Galactic Magnetic Fields from Superconducting Strings

C. J. A. P. Martins[*] and E. P. S. Shellard[†]

Department of Applied Mathematics and Theoretical Physics
University of Cambridge
Silver Street, Cambridge CB3 9EW, U.K.

(23 May 1997)

We use a simple analytic model for the evolution of currents in superconducting strings to estimate the strength of the 'seed' magnetic fields generated by these strings. This model is an extension of the evolution model of Martins and Shellard depending on a parameter f which characterizes the importance of equilibration process in the evolution of the currents. For GUT-scale strings, we find that a viable seed magnetic field for the galactic dynamo can be generated if equilibration is weak. On the other hand, electroweak-scale strings originate magnetic fields that are smaller than required.

98.80.Cq, 11.27.+d, 98.62.En

I. INTRODUCTION

All observations show that our galaxy, together with a good number of other spirals (and galaxy clusters in the inter-cluster medium), possess 'regular' magnetic fields with magnitude $B \sim 10^{-6}\,G$, on scales of several kiloparsecs [1–3] (in addition, there is a small-scale random component in our galaxy with the same magnitude and a coherence length of about $100\,Mpc$). No magnetic fields have been observed on larger scales, current observational bounds (obtained from the analysis of remote radio galaxies and quasars) being about $B < 10^{-9}\,G$ [4]—but it should be said that, since one needs to separate between source, Galaxy and intergalactic contributions, these are quite difficult observations. Even though these magnatic fields are fairly small, it is of course possible to find localized objects with much larger magnetic fields—for example, X-ray sources near neutron stars can have $B \sim 10^{13}\,G$.

These galactic fields are associated with the interstellar gas. Stellar magnetic fields are known to be extremely small between stars, and in any case they could not explain the observed large-scale structure. Even though the magnetic fields do not play any significant part in the equilibrium and dynamics of the galaxy, they do have a significant role in the propagation of cosmic rays, gasdynamical processes—notably star formation—and in the mechanism by which cosmic dust is oriented. In particular, star formation is not possible without a magnetic field—its role being that of transporting angular momentum outwards so that the collapse of the proto-stellar cloud can continue.

The large coherence scales of these magnetic fields (several kiloparsecs) means that it is difficult to find mechanisms capable of createing them (for example, thermal, chemical or other 'battery' effects are inadequate). Thus, even though a large number of possibilities have been considered in the past—including vorticity [5], inflationary models [6,7] and cosmological phase transitions [8]—none seems to be particularly compelling.

In this paper we consider the possibility of the galactic magnetic fields [1–3] being generated by superconducting [9,10] cosmic strings. Our discussion is based on the quantitative evolution model of Martins and Shellard [11,12], together with a simple 'toy model' for the evolution of the superconducting currents [13,14]. We consider both electroweak- and GUT-scale cosmic strings. While earlier estimates indicated that superconducting GUT strings were observationally ruled out, since they led to unacceptebly large densities of springs and vortons [15], it has been shown—by Peter [16] for the former, by Martins and Shellard [14] for the later—that neither of these form in general.

The plan of this paper is as follows. In section II we review some basic notions about astrophysical and cosmological magnetic fields. Following this we briefly review our evolution model (first discussed in [13], see also [14]) in section III and analise its solutions. In section IV we determine the relevant 'seed' magnetic fields and compare our results with existing bounds; finally (section V), we discuss the relevance of our results.

II. ASTROPHYSICAL MAGNETIC FIELDS

It has been shown [1–3] that in order to explain the observed galactic magnetic fields $B_o \sim 10^{-6}\,G$ one needs a seed field $B_s \geq 10^{-19}\,G$ on the comoving scale of a protogalaxy (about $100\,kpc$)—such field can then be amplified, by a dynamo mechanism, to the observationally required value. Since the gravitational collapse of the protogalaxies enhances any frozen-in magnetic field, this seed field corresponds to an rms field

$$B_g \geq 10^{-22}\,G \tag{2.1}$$

at the epoch when galactic scales $d_g \sim 1\,Mpc$ fall inside the horizon (that is, at a time $t_g \sim 2.5 \times 10^{-3}\,t_{eq}$). On the other hand, one would need $B_g \sim 10^{-10}G$ to create significant magnetic fields through adiabatic compression alone.

We should recall that there are also upper bounds on cosmologically interesting magnetic fields. Firstly, primordial nucleosynthesis is sensitive to magnetic fields, which change the expansion rate of the universe an consequently the rates of the reactions that produce the light elements. This gives rise to a bound [17]

$$B_{nuc} \leq 10^9\,G\,, \tag{2.2}$$

or equivalently

$$\left(\frac{\rho_B}{\rho_\gamma}\right)_{nuc} \leq 0.28 \tag{2.3}$$

at the nucleosynthesis epoch. Secondly, there are bounds on the strength of a uniform 'primordial' magnetic field; as we pointed out in the previous section, analysis of radio galaxies and quasars yields the constraint [4]

$$B_0 \leq 10^{-9}\,G\,. \tag{2.4}$$

A more recent analysis of the consequences of primordial magnetic fields for the cosmic microwave background [18] produces a comparable bound

$$B_0 \leq 6.8 \times 10^{-9}\Omega_0^{1/2}h\,G\,; \tag{2.5}$$

in terms of densities, these can be written

$$\left(\frac{\rho_B}{\rho_\gamma}\right)_0 \leq 10^{-7}\,. \tag{2.6}$$

It should be pointed out that an *ab initio* uniform magnetic field does not violate homogeneity, but it does make the cosmological expansion anisotropic. It is thus much more appealing to assume that the required magnetic seed fields were generated by some dynamical process. The currently favoured paradigm is dynamo theory [1,2], which develops on Larmor's suggestion [19] that it is possible to excite magnetic fields by the motion of a conductive fluid in a gas, and allows energy associated with the differential rotation of spiral galaxies to be converted into magnetic field energy. In this model the state of the magnetic field today is almost independent of initial conditions: a dynamo process results in equipartition of energy between the plasma kinetic and magnetic energies on scales up to the coherence length of the field.

The first proposal for the origin of the required seed field was originally due to Harrison [5], and subsequently developed by Mishustin and Ruzmaiakin [20]. This claims that the relative motion of protons and electrons induced by vorticity present before the epoch of decoupling (the electrons in vortices being more strongly coupled to the background radiation than the protons) produces primeval currents and hence magnetic fields. Obviously, this requires a source of vorticity.

Later Vachaspati and Vilenkin [21] have suggested that strings with small-scale structure are such a source. They pointed out that, since the matter flow in baryonic wakes is turbulent, velocity gradients will be induced in the flow by the small-scale wiggles, which produces the required vorticity. Avelino and Shellard [22] have also shown that dynamical friction between cosmic strings and matter provides a further source of vorticity.

It is also possible to generate large-scale magnetic fields at the end of an inflationary epoch [6,7]. However, these models generally need to invoke rather speculative changes to the nature of the electromagnetic interactions during the inflationary epoch, whose only motivation seems to be the generation of such magnetic fields.

Still, none of the above (or other) possibilities provides a compelling mechanism for the generation of a magnetic field with strength B_g on galactic scales d_g today. We should also point out that there is evidence that magnetic fields were present in moderately young galaxies (at redshifts $z \sim 1$–2) [3]. This is a challenge to dynamo theory, in that at least the simplest galactic dynamo models cannot generate micro-Gauss strength magnetic fields at such early epochs [23].

III. EVOLUTION OF THE STRING NETWORK

Due to the strings' statistical nature, analytic evolution methods must be 'thermodynamic', that is one must describe the network by a small number of macroscopic (or 'averaged') quantities whose evolution equations are derived from the microscopic string equations of motion. The first such model providing a quantitative picture of the complete evolution of a string network (and the corresponding loop population) has been developed by Martins and Shellard (see [11,12] for a detailed analysis of the model), and has two such quantities, the long-string correlation length $\rho_\infty \equiv \mu/L^2$ (μ being the string mass per unit length) and the string RMS velocity, $v^2 \equiv \langle \dot{\mathbf{x}}^2 \rangle$. It also includes two 'phenomenological' parameters, a 'loop chopping efficiency' $0 < \tilde{c} < 1/2$ and a 'small-scale structure parameter' $0 < k < 1$. These are sufficient to quantitatively describe the large-scale properties of a cosmic string network.

More recently, this has been extended with a 'toy model' for the evolution of the superconducting currents (see [13,14]). Assuming that there is a 'superconducting correlation length', denoted ξ, which measures the scale over which one has coherent current and charge densities on the strings, we can define N to be the number of uncorrelated current regions (in the long-string network) in a co-moving volume V. It is then fairly straightforward to see how the dynamics of the string network affects N and obtain an evolution equation for it. The only non-trivial issue is that of the dynamics of the currents themselves. There is evidence that some kind of 'equilibration' process acts between neighbouring current regions, counteracting the creation of new regions by inter-commutings and helping their removal by loops. Notably, the simulations of Laguna and Matzner [24] show that as the result of inter-commutings charges pile up at current discontinuities and move with the kinks, but their strength decreases with time. Also, Austin, Copeland and Kibble have shown [25] that in an expanding universe correlations between left- and right-moving modes develop due both to stretching and inter-commuting (particularly when loops form). We model this term by assuming that after each Hubble time, a fraction f of the N regions existing at its start will have equilibrated with one of its neighbours,

$$\left(\frac{dN}{dt}\right)_{dynamics} = -fHN \, ; \tag{3.1}$$

note that new regions are obviously created by inter-commuting during the Hubble time in question, so that f can be larger than unity. Alternatively we can say that for a given f, the number of regions that were present in a given volume at a time t will have disappeared due to equilibration at a time $t + (fH)^{-1}$.

We therefore obtain the following evolution equation for N

$$\frac{dN}{dt} = G\left(\frac{\ell}{\xi}\right) \frac{v_\infty}{\alpha} \frac{V}{L^4} - fHN \, , \tag{3.2}$$

where the 'correction factor' G has the form (see [14] for a complete discussion)

$$G\left(\frac{\ell}{\xi}\right) = \begin{cases} 2 - \tilde{c}\left(\frac{\ell}{\xi} + 2\right) \, , & \frac{\ell}{\xi} > 1 \\ 2(1 - 2\tilde{c})\alpha + (2 - 3\tilde{c} - 2\alpha + 4\tilde{c}\alpha)\frac{\ell}{\xi} \, , & \frac{\ell}{\xi} \le 1 \end{cases} \; ; \tag{3.3}$$

loops are assumed to form with a size $\ell(t) = \alpha(t)L(t)$, where $\alpha \sim 1$ while the string network is is the friction-dominated epoch and $\alpha = \alpha_{sc} \ll 1$ once it has reached the linear scaling regime (see [12]).

For what follows it is more convenient to introduce N_L, defined to be the number of uncorrelated current regions per long-string correlation length,

$$N_L \equiv \frac{L}{\xi} \, ; \tag{3.4}$$

in terms of N_L, (3.2) has the form

$$\frac{dN_L}{dt} = (3v_\infty^2 - f)HN_L + \frac{3}{2}\frac{v_\infty^2}{\ell_{\rm f}}N_L + \left(\frac{1}{\alpha}G(\alpha N_L) + \frac{3}{2}\tilde{c}N_L\right)\frac{v_\infty}{L} \, , \tag{3.5}$$

where $\ell_{\rm f}$ is the friction lengthscale due to particle scattering off strings. This has been shown [13] to be the dominant friction mechanism, except possibly if there are background magnetic fields (in which case plasma friction effects would be more important). Such possibility will not be considered in this paper, since we are interested in the magnetic fields generated by the strings themselves. Note that to obtain this equation one needs to use the evolution equation for the long-string correlation length L, and that one can equivalently define G as

3

$$G\left(\alpha N_L\right) = \begin{cases} 2 - \tilde{c}\left(\alpha N_L + 2\right), & \alpha N_L > 1 \\ 2(1 - 2\tilde{c})\alpha + (2 - 3\tilde{c} - 2\alpha + 4\tilde{c}\alpha)\alpha N_L, & \alpha N_L \le 1 \end{cases} . \tag{3.6}$$

Now the question is, of course, what is f. From a more intuitive point of view, an equivalent question is the following: given a particular piece of string with a given current, is it more likely to disappear from the network by this equilibration mechanism or by being incorporated in a loop? Even though a precise answer can probably only be given by means of a numerical simulation, some physical arguments can be used to constrain it [13,14]. In the present paper, however, we will postpone this interesting discussion and treat f as a free parameter. We simply point out that, according to our previous results [13,14], if equilibration is inexistent or ineffective, then N_L grows without bound at late times, whereas if equilibration is effective N_L eventually becomes a constant (which corresponds to linear scaling of ξ).

We should also say at this stage that once the network leaves the friction-dominated regime and strings become relativistic other mechanisms (notably radiation) can cause charge losses in the long strings (as well as in loops). Thus we do not expect our toy model to provide quantitatively correct answers, but we do expect it to provide reliable order-of-magnitude estimates.

IV. MAGNETIC FIELDS

We expect the seed magnetic fields from cosmic strings to be coherent on the scale at which loops are being formed, that is $\ell(t) = \alpha(t)L(t)$. Hence if N_L is the number of uncorrelated regions per long-string correlation length and $L(t) = \gamma(t)t$ we find

$$B_g = \frac{2\pi e}{c^2 t_g^2 \alpha^{3/2}} \frac{N_L^{1/2}}{\gamma^2}, \tag{4.1}$$

and all we have to do is evaluate N_L and γ using our analytic model, while checking that at the nucleosynthesis epoch the corresponding magnetic fields are consistent with existing bounds—which is isdeed the case.

In figures 1 and 2 we plot the expected coherent seed fields at the epoch t_g, for $0 \le f \le 8$—as can be seen, for large enough f the result is almost independent of it. It can be seen that if equilibration is ineffective electroweak strings just fall short of producing the required seed fields, $B_{seed} \sim 10^{-22} \, Gauss$, but GUT-scale strings can in the same circumstances produce such fields—all we require is an ineffective equilibration mechanism, $f \le 0.5$.

Note that there is almost no dependence on initial conditions in the GUT case, but such dependence persists for the electroweak string network if equilibration is weak. this is because the GUT-scale string network is in the linear scaling regime at t_g, while the electroweak network is in the Kibble regime (see [12] for a detailed description of these regimes). In the GUT case, N_L is constant (that is, ξ is caling linearly) provided $f \ge 1.88$ (see [13,14]), whereas if $f = 0$ ξ is constant and N_L is growing linearly. This explains the large differences between the magnetic fields at high and low f in the GUT case compared to the electroweak one. On the other hand, $t_g \sim 2.5 \times 10^{47} \, t_{GUT}$ and $t_g \sim 2.5 \times 10^{19} \, t_{EW}$ in the GUT and electroweak cases respectively. Thus, despite evolving for a much shorter time, electroweak strings are friction dominated much longer than GUT-ones, and so if equilibration is effective they can build-up much larger currents. This is the reason why in this case electroweak strings generate much larger magnetic fields.

V. CONCLUSIONS

In this paper we have used the quantitative string evolution model of Martins and Shellard [11,12], together with a simple toy model for the evolution of currents on the strings [13,14] to study the possibilty of using superconducting strings to provide the 'seed' galactic magnetic fields.

We have shown that GUT-scale superconducting strings can provide the required fields for the galactic dynamo mechanism provided that current equilibration mechanism are ineffective, while similar fields from electroweak strings are too weak.

Clearly, the outstanding issue, in this and other cosmological scenarios involving superconducting cosmic strings, is that of the importance of charge and current equilibration mechanisms on the strings, and a more detailed study of it is therefore required.

[1] Ya. B. Zel'dovich, A. A. Ruzmaiakin & D. D. Sokoloff, *'Magnetic Fields in Astrophysics'*, Gordon and Breach, New York (1983).

[2] A. A. Ruzmaiakin, A. M. Shukurov & D. D. Sokoloff, *'Magnetic Fields of Galaxies'*, Kluwer, Dordrecht (1988).

[3] P. P. Kronberg, Rep. Prog. Phys. **57**, 325 (1994);
E. G. Zweibel & C. Heiles, Nature **385**, 131 (1997).

[4] J. P. Vollée, Astrophys. Lett. **23**, 85 (1983).

[5] E. R. Harrison, M. N. R. A. S. **147**, 279 (1970).

[6] M. S. Turner & L. W. Widrow, Phys. Rev. **D37**, 2743 (1988);
B. Ratra, Ap. J. **391**, L1 (1992).

[7] J. Quashnock, A. Loeb & D. N. Spergel, Ap. J. Lett. **344**, 9 (1989);
A. D. Dolgov, J. Silk, Phys. Rev. **D47**, 3144 (1993);
A. D. Dolgov, Phys. Rev. **D48**, 2499 (1993).

[8] C. J. Hogan, Phys. Rev. Lett. **51**, 1488 (1983);
T. Vachaspati, Phys. Lett. **B265**, 258 (1991);
K. Enqvist & P. Olesen, Phys. Lett. **B319**, 178 (1993).

[9] E. Witten, Nucl. Phys. **B249**, 557 (1985).

[10] A. Vilenkin & E. P. S. Shellard, *'Cosmic Strings and other Topological Defects'*, Cambridge University Press (1994).

[11] C. J. A. P. Martins & E. P. S. Shellard, Phys. Rev. **D53**, 575 (1996).

[12] C. J. A. P. Martins & E. P. S. Shellard, Phys. Rev. **D54**, 2535 (1996).

[13] C. J. A. P. Martins & E. P. S. Shellard, 'Evolution of Superconducting String Currents', astro-ph/9706286, submitted to Phys. Lett. **B**.

[14] C. J. A. P. Martins & E. P. S. Shellard, 'Conditions for Vorton Formation', submitted to Phys. Rev. **D**.

[15] R. L. Davis & E. P. S. Shellard, Nucl. Phys. **B249**, 557 (1989).

[16] P. Peter, Phys. Rev. **D47**, 3169 (1993).

[17] B. Cheng, D. N. Schramm & J. W. Truran, Phys. Rev. **D49**, 5006 (1994);
B. Cheng *et al.*, Phys. Rev. **D54**, 4714 (1996).

[18] J. D. Barrow, P. G. Ferreira & J. Silk, Phys. Rev. Lett. **78**, 3610 (1997).

[19] J. Larmor, *in 'Report of the 87th Meeting of the British Association for the Advancement of Science'*, John Murray, London (1919).

[20] I. N. Mishustin & A. A. Ruzmaiakin, J. E. T. P. **61**, 441 (1971);
I. N. Mishustin & A. A. Ruzmaiakin, Sov. Phys. JETP **34**, 223 (1972).

[21] T. Vachaspati & A. Vilenkin, Phys. Rev. Lett. **67**, 1057 (1991).

[22] P. P. Avelino & E. P. S. Shellard, Phys. Rev. **D51**, 5946 (1995).

[23] A. M. Wolfe, K. Lanzetta & A. L. Oren, Ap. J. **388**, 17 (1992).

[24] P. Laguna & R. A. Matzner, Phys. Rev. **D41**, 1751 (1990).

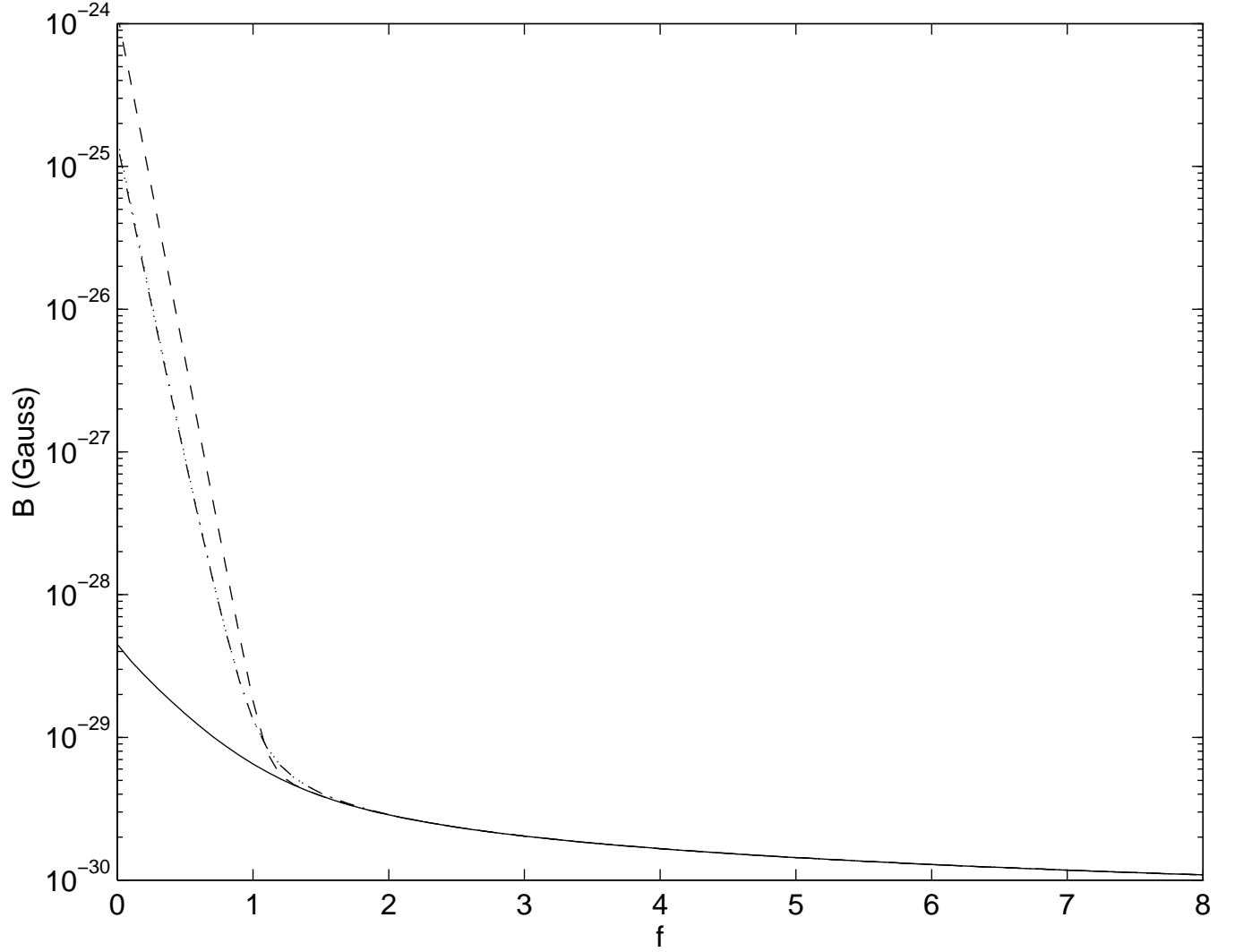[25] D. Austin, E. J. Copeland and T. W. B. Kibble, Phys. Rev. **D48**, 5594 (1993).

FIG. 1. The magnitude of coherent magnetic fields, at the epoch when galaxy scales fall inside the horizon, as a function of f for electroweak-scale superconducting string networks. The different lines correspond to initial conditions typical of string-forming and superconducting phase transitions that are respectively of 1st & 1st (solid), 1st & 2nd (dashed), 2nd & 1st (dash-dotted) and 2nd & 2nd (dotted) order.
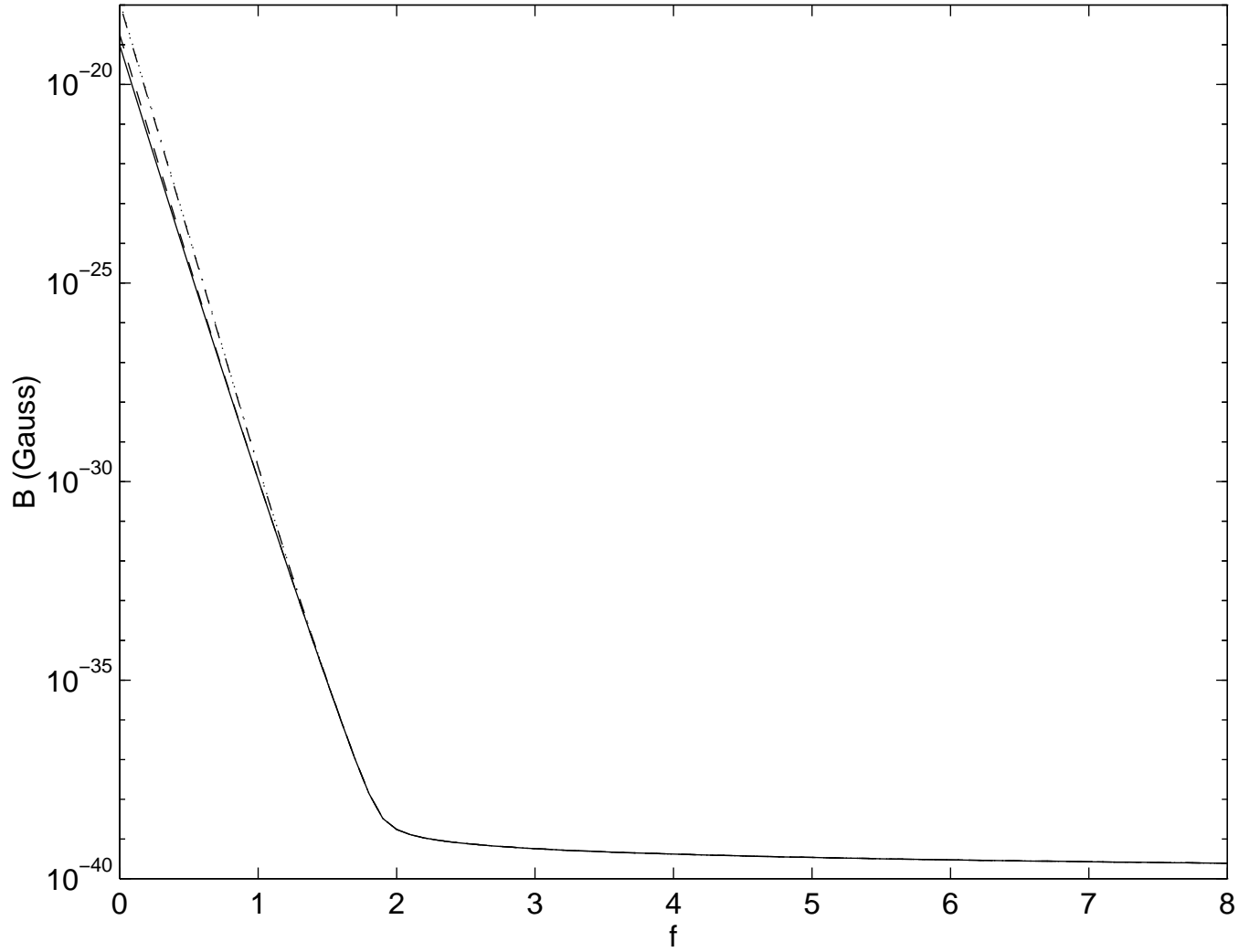
FIG. 2. The magnitude of coherent magnetic fields, at the epoch when galaxy scales fall inside the horizon, as a function of f for GUT-scale superconducting string networks. The different lines correspond to initial conditions typical of string-forming and superconducting phase transitions that are respectively of 1st & 1st (solid), 1st & 2nd (dashed), 2nd & 1st (dash-dotted) and 2nd & 2nd (dotted) order.